VESSEL VOLUME CALCULATOR

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Contents

Introduction

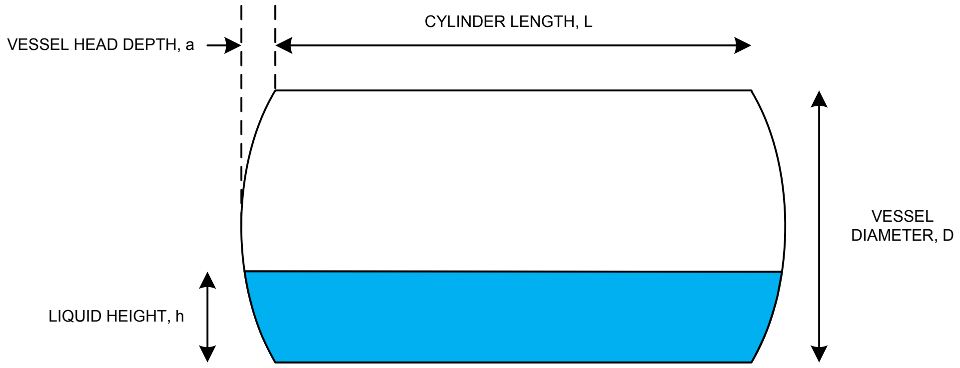
This document describes the basis and operation of the Blackmonk Engineering Vessel Volume Calculator.

The calculation methodology is based on that described in “Computing Fluid Tank Volumes” Chemical Processing, November 17, 2002, written by Dan Jones.

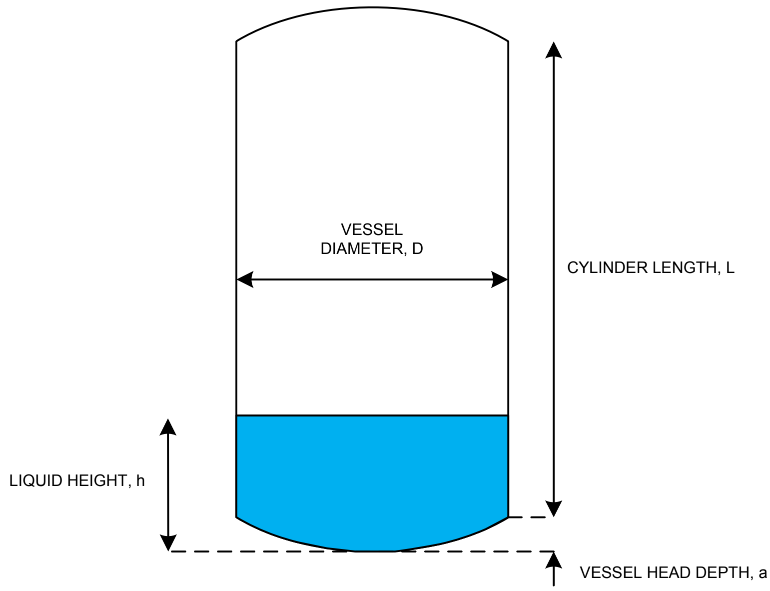
The calculator determines the volume occupied by a specified height of liquid in a cylindrical horizontal or vertical vessel with conical, elliptical, flat, guppy (horizontal tanks only), torispherical or spherical heads. The vessel diameter and cylindrical (tan to tan) length are required inputs.

The calculator is applicable to full and partially full vessels. For horizontal vessels the heads are assumed to be identical. Only the bottom head is considered for a vertical vessel.

System Diagram



HORIZONTAL VESSEL



VERTICAL VESSEL

Calculation Inputs

The following parameters are user specified inputs to the calculation:

Input	Description	Units
Cylinder length	Mandatory user specified length of cylindrical section of vessel (tan to tan)	m
Vessel diameter	Mandatory user specified vessel diameter	m
Vessel head depth	Mandatory user specified vessel head depth (internal head depth). Note this input is not used in calculating torispherical head volumes	m
Liquid height	Mandatory user specified height of liquid above the lowest point of vessel	m
Vessel orientation	Mandatory user specified vessel orientation. Can be horizontal or vertical	N/A
Vessel head type	Mandatory user specified vessel head type. Can be conical, elliptical, flat, guppy (horizontal vessels only), torispherical or spherical	N/A
Knuckle radius parameter	Mandatory user specified knuckle radius parameter. Note this input is only used in calculating torispherical head volumes	N/A
Dish radius parameter	Mandatory user specified dish radius parameter. Note this input is only used in calculating torispherical head volumes	N/A

Calculation Output

The following parameter is calculated by the software and displayed to the user:

Output	Description	Units
Total liquid volume in vessel	Total liquid volume in vessel	m^3

Horizontal Vessel Volumes

Fluid volume as a function of fluid height can be calculated for a horizontal cylindrical tank with either conical, ellipsoidal, guppy, spherical, or torispherical heads where the fluid height, h, is measured from the tank bottom to the fluid surface, see Figs. 1 and 2. A guppy head is a conical head where the apex of the conical head is level with the top of the cylindrical section of the tank as shown in Fig. 1. A torispherical head is an ASME-type head defined by a knuckle-radius parameter, k, and a dish-radius parameter, f, as shown in Fig. 2.

An ellipsoidal head must be exactly half of an ellipsoid of revolution; only a hemiellipsoid is valid – no “segment” of an ellipsoid will work as is true in the case of a spherical head where the head may be a spherical segment. For a spherical head, $|a| \le R$, where R is the radius of the cylindrical tank body. Where concave conical, ellipsoidal, guppy, spherical, or torispherical heads are considered, then $|a| \le L/2$.

Both heads of a horizontal cylindrical tank must be identical for the equations to work; i.e., if one head is conical, the other must be conical with the same dimensions. However, the equations can be combined to deal with fluid volume calculations of horizontal tanks with heads of different shapes. For instance, if a horizontal cylindrical tank has a conical head on one end and an ellipsoidal head on the other end, calculate fluid volumes of two tanks, one with conical heads and the other with ellipsoidal heads, and average the results to get the desired fluid volume. The heads of a horizontal tank may be flat ($a = 0$), convex ($a > 0$), or concave ($a < 0$).

The following variables must be within the ranges stated:

- $|a| \le R$ for spherical heads
- $|a| \le L/2$ for concave ends
- $0 \le h \le 2R$ for all tanks
- $f > 0.5$ for torispherical heads
- $0 \le k \le 0.5$ for torispherical heads
- $D > 0$
- $L \ge 0$

Horizontal Vessel Equations

The specific equations for fluid volumes in horizontal cylindrical tanks with conical, ellipsoidal, guppy, spherical, and torispherical heads are given below (use radian angular measure for all trigonometric functions, and $D/2 = R > 0$ for all equations):

Conical heads

$$V_f = A_f L + \frac{2aR^2}{3} \times \begin{cases} K & \ldots\ldots\ldots\ldots \quad 0 \le h < R \\ \pi/2 & \ldots\ldots\ldots \quad h = R \\ \pi - K & \ldots\ldots\ldots \quad R < h \le 2R \end{cases}$$

$$K \equiv \cos^{-1} M + M^3 \cosh^{-1}\frac{1}{M} - 2M\sqrt{1-M^2} \qquad\qquad M = \left|\frac{R-h}{R}\right|$$

Ellipsoidal heads

$$V_f = A_f L + \pi a h^2 \left(1 - \frac{h}{3R}\right)$$

Guppy heads

$$V_f = A_f L + \frac{2aR^2}{3}\cos^{-1}\left(1-\frac{h}{R}\right) + \frac{2a}{9R}\sqrt{2Rh-h^2}\,(2h-3R)(h+R)$$

Spherical heads

$$V_f = A_f L + \begin{cases} \dfrac{\pi a}{6}\left(3R^2 + a^2\right) & \ldots\ldots\ldots\ldots \quad h = R, \quad |a| \le R \\ \dfrac{\pi a}{3}\left(3R^2 + a^2\right) & \ldots\ldots\ldots\ldots \quad h = D, \quad |a| \le R \\ \pi a h^2\left(1 - \dfrac{h}{3R}\right) & \ldots\ldots\ldots\ldots \quad h = 0 \text{ or } a = 0, R, -R \\ \dfrac{a}{|a|}\left\{\dfrac{2r^3}{3}\left[\cos^{-1}\dfrac{R^2 - rw}{R(w-r)} + \cos^{-1}\dfrac{R^2 + rw}{R(w+r)} - \dfrac{z}{r}\left(2 + \left(\dfrac{R}{r}\right)^2\right)\cos^{-1}\dfrac{w}{R}\right]\right. & \\ \qquad \left. - 2\left(wr^2 - \dfrac{w^3}{3}\right)\tan^{-1}\dfrac{y}{z} + \dfrac{4wyz}{3}\right\} & \ldots\ldots\ldots \quad h \ne R, D; \quad a \ne 0, R, -R; \quad |a| \ge 0.01D \\ \dfrac{a}{|a|}\left[2\displaystyle\int_w^R \left(r^2 - x^2\right)\tan^{-1}\sqrt{\dfrac{R^2 - x^2}{r^2 - R^2}}\,dx - A_f z\right] & \ldots\ldots\ldots \quad h \ne R, D; \quad a \ne 0, R, -R; \quad |a| < 0.01D \end{cases}$$

$$r = \frac{a^2 + R^2}{2|a|} \quad a \ne 0 \quad ; \qquad a = \pm\left(r - \sqrt{r^2 - R^2}\right) \quad +(-) \text{ for convex (concave) heads}$$

$$w \equiv R - h \qquad\qquad y \equiv \sqrt{2Rh - h^2} \qquad\qquad z \equiv \sqrt{r^2 - R^2}$$

Torispherical heads

In the V_f equation, use +(-) for convex(concave) heads.

$$V_f = A_f L \pm \begin{cases} 2\, v_1 & \ldots\ldots\ 0 \le h \le h_1 \\ 2\left(v_{1,max} + v_2 + v_3\right) & \ldots\ldots\ h_1 < h < h_2 \\ 2\left[2 v_{1,max} - v_1(h = D - h) + v_{2,max} + v_{3,max}\right] & \ldots\ldots\ h_2 \le h \le D \end{cases}$$

$$v_1 \equiv \int_0^{\sqrt{2kDh-h^2}} \left[n^2 \sin^{-1}\frac{\sqrt{n^2 - w^2}}{n} - w\sqrt{n^2 - w^2} \right] dx$$

$$v_2 \equiv \int_0^{kD\cos\alpha} \left[n^2 \left(\cos^{-1}\frac{w}{n} - \cos^{-1}\frac{g}{n} \right) - w\sqrt{n^2 - w^2} + g\sqrt{n^2 - g^2} \right] dx$$

$$v_3 \equiv \begin{cases} \dfrac{r^3}{3}\left[\cos^{-1}\dfrac{g^2 - r w}{g(w - r)} + \cos^{-1}\dfrac{g^2 + r w}{g(w + r)} - \dfrac{z}{r}\left(2 + \left(\dfrac{g}{r}\right)^2 \right)\cos^{-1}\dfrac{w}{g} \right] - \left(w r^2 - \dfrac{w^3}{3} \right)\tan^{-1}\dfrac{\sqrt{g^2 - w^2}}{z} \\ \qquad + \dfrac{w z\sqrt{g^2 - w^2}}{6} + \dfrac{wz}{2}\sqrt{2g(h - h_1) - (h - h_1)^2} & \ldots\ldots\ 0.5 < f \le 10 \\ \displaystyle\int_w^g \left(r^2 - x^2\right)\tan^{-1}\frac{\sqrt{g^2 - x^2}}{z}\, dx - \frac{z}{2}\left(g^2\cos^{-1}\frac{w}{g} - w\sqrt{2g(h - h_1) - (h - h_1)^2} \right) & \ldots\ldots\ 0.5 < f < 10{,}000 \end{cases}$$

$$v_{1,max} \equiv v_1(h = h_1)$$

$$v_{2,max} \equiv v_2(h = h_2)$$

$$v_{3,max} \equiv v_3(h = h_2) = \frac{\pi a_1}{6}\left(3g^2 + a_1^2\right) \qquad a_1 \equiv r(1 - \cos\alpha)$$

$$\alpha \equiv \sin^{-1}\frac{1 - 2k}{2(f - k)} = \cos^{-1}\frac{\sqrt{4f^2 - 8fk + 4k - 1}}{2(f - k)} \qquad r \equiv fD$$

$$h_1 \equiv kD(1 - \sin\alpha) \qquad h_2 \equiv D - h_1$$

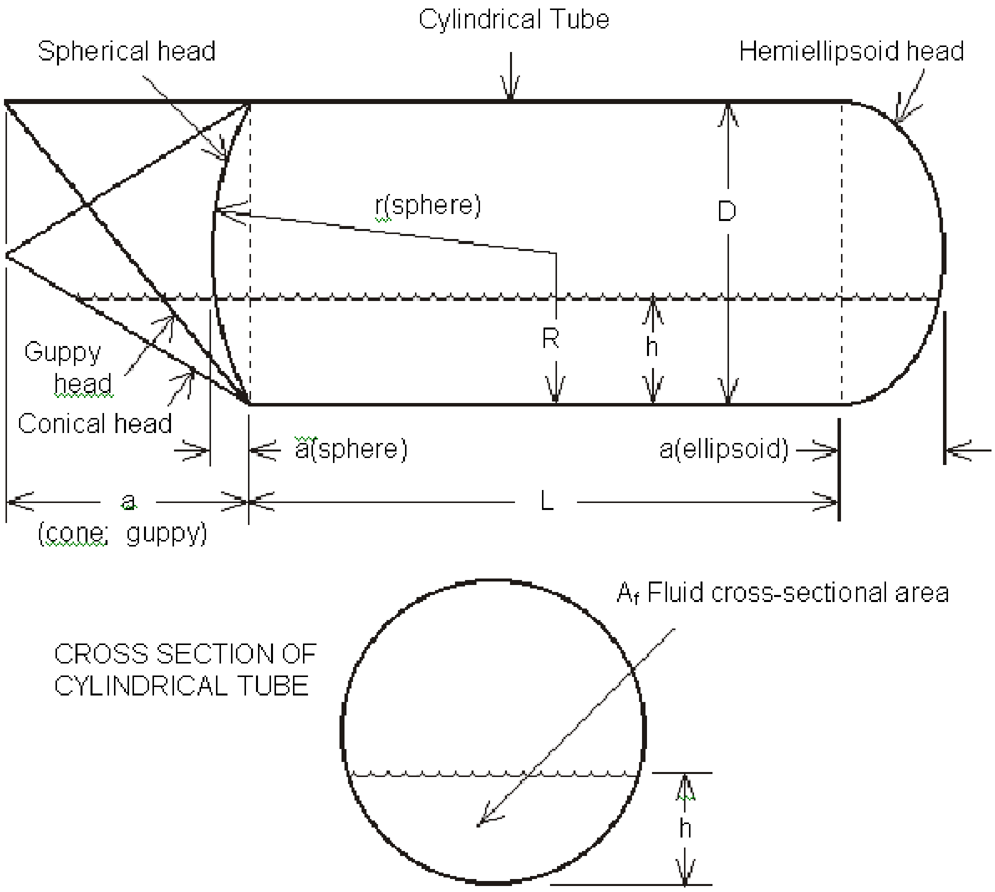
$$n \equiv R - kD + \sqrt{k^2D^2 - x^2} \qquad w \equiv R - h$$

$$g \equiv fD\sin\alpha = r\sin\alpha \qquad z \equiv \sqrt{r^2 - g^2} = fD\cos\alpha = r\cos\alpha$$

In the above equations, V_f is the total volume of fluid in the tank in cubic units consistent with the linear units of tank dimension parameters, and A_f is the cross-sectional area of fluid in the cylindrical body of the tank in square units consistent with the linear units used for R and h. The equation for A_f is given by:

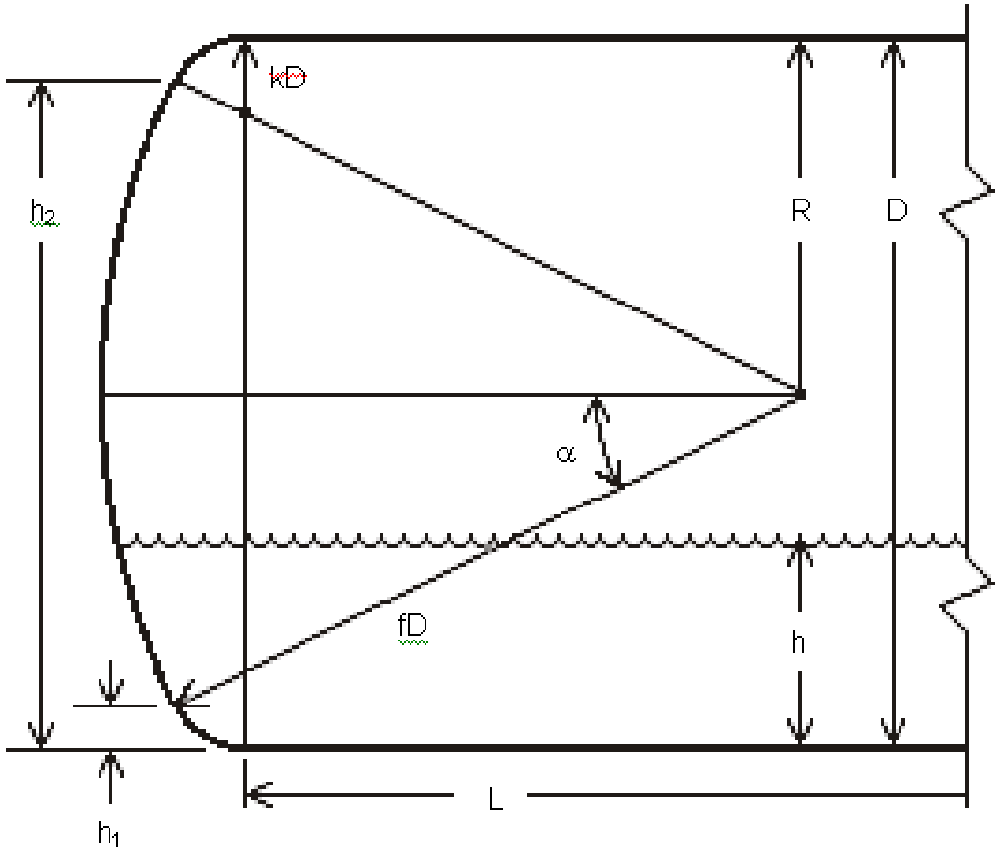
$$A_f = R^2\cos^{-1}\left(\frac{R - h}{R}\right) - (R - h)\sqrt{2Rh - h^2}$$

Figure 1: Parameters for Horizontal Cylindrical Tanks with Conical, Ellipsoidal, Guppy or Spherical Heads



1. Both heads of a tank must be identical. Above diagram is for definition of parameters only.
2. Cylindrical tube of diameter D ($D > 0$), radius R ($R > 0$), and length L ($L \geq 0$).
3. For spherical head of radius r, $r \geq R$ and $|a| \leq R$.
4. For convex head other than spherical, $0 < a < \infty$, for concave head $a < 0$.
5. $L \geq 0$ for $a \geq 0$, $L \geq 2|a|$ for $a < 0$.
6. Ellipsoidal head must be exactly half of an ellipsoid of revolution.
7. $0 \leq h \leq D$.

Figure 2: Parameters for Horizontal Cylindrical Tanks with Torispherical Heads



Solution of Integrals for Spherical and Torispherical Head Equations

The calculator uses Simpson's Rule to numerically integrate the integrals required to determine the spherical and torispherical head volumes.

Each integral is evaluated at 11 points starting at the lower limit of integration and ending at the upper limit of integration.

Vertical Vessel Volumes

Fluid volume in a vertical cylindrical tank with either a conical, ellipsoidal, spherical, or torispherical bottom can be calculated, where the fluid height, h, is measured from the centre of the bottom of the tank to the surface of the fluid in the tank. See Figures 3 and 4 for tank configurations and dimension parameters.

A torispherical bottom is an ASME-type bottom defined by a knuckle-radius factor and a dish-radius factor as shown graphically in Fig. 4. The knuckle radius will then be kD and the dish radius will be fD. An ellipsoidal bottom must be exactly half of an ellipsoid of revolution. For a spherical bottom, |a| ≤

R, where a is the depth of the spherical bottom and R is the radius of the cylindrical section of the tank.

The following parameter ranges must be observed:

- $a \geq 0$ for all vertical tanks, $a \leq R$ for a spherical bottom
- $f > 0.5$ for a torispherical bottom
- $0 \leq k \leq 0.5$ for a torispherical bottom
- $D > 0$

Vertical Vessel Equations

The specific equations for fluid volumes in vertical cylindrical tanks with conical, ellipsoidal, spherical, and torispherical bottoms are given below (use radian angular measure for all trigonometric functions, and D > 0 for all equations):

Conical bottom

$$V_f = \begin{cases} \dfrac{\pi}{4}\left(\dfrac{Dh}{a}\right)^2\left(\dfrac{h}{3}\right) & \text{..............................} \quad h < a \\ \dfrac{\pi D^2}{4}\left(h - \dfrac{2a}{3}\right) & \text{.............................} \quad h \geq a \end{cases}$$

Ellipsoidal bottom

$$V_f = \begin{cases} \dfrac{\pi}{4}\left(\dfrac{Dh}{a}\right)^2\left(a - \dfrac{h}{3}\right) & \text{........................} \quad h < a \\ \dfrac{\pi D^2}{4}\left(h - \dfrac{a}{3}\right) & \text{..............................} \quad h \geq a \end{cases}$$

Spherical bottom

$$V_f = \begin{cases} \dfrac{\pi h^2}{4}\left(2a + \dfrac{D^2}{2a} - \dfrac{4h}{3}\right) & \text{................} \quad h < a \quad ; \quad (a \leq D/2) \\ \dfrac{\pi}{4}\left(\dfrac{2a^3}{3} - \dfrac{aD^2}{2} + hD^2\right) & \text{..............} \quad h \geq a \quad ; \quad (a \leq D/2) \end{cases}$$

Torispherical bottom

$$
V_f = \begin{cases}
\dfrac{\pi h^2}{4}\left(2a_1 + \dfrac{D_1^2}{2a_1} - \dfrac{4h}{3}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 0 \le h \le a_1 \\
\dfrac{\pi}{4}\left(\dfrac{2a_1^3}{3} + \dfrac{a_1 D_1^2}{2}\right) + \pi u\left[\left(\dfrac{D}{2} - kD\right)^2 + s\right] + \dfrac{\pi t u^2}{2} - \dfrac{\pi u^3}{3} \\
\qquad + \pi D(1-2k)\left[\dfrac{2u-t}{4}\sqrt{s + tu - u^2} + \dfrac{t\sqrt{s}}{4} + \dfrac{k^2 D^2}{2}\left(\cos^{-1}\dfrac{t-2u}{2kD} - \alpha\right)\right] \quad \ldots\ldots \quad a_1 < h \le a_1 + a_2 \\
\dfrac{\pi}{4}\left(\dfrac{2a_1^3}{3} + \dfrac{a_1 D_1^2}{2}\right) + \dfrac{\pi t}{2}\left[\left(\dfrac{D}{2} - kD\right)^2 + s\right] \\
\qquad + \dfrac{\pi t^3}{12} + \pi D(1-2k)\left(\dfrac{t\sqrt{s}}{4} + \dfrac{k^2 D^2}{2}\sin^{-1}(\cos\alpha)\right) + \dfrac{\pi D^2}{4}\left[h - (a_1 + a_2)\right] \quad \ldots\ldots \quad a_1 + a_2 < h
\end{cases}
$$

$$\alpha \equiv \sin^{-1}\frac{1-2k}{2(f-k)} = \cos^{-1}\frac{\sqrt{4f^2 - 8fk + 4k - 1}}{2(f-k)}$$

$$a_1 \equiv fD(1-\cos\alpha)$$
$$a_2 \equiv kD\cos\alpha$$
$$D_1 \equiv 2fD\sin\alpha$$
$$s \equiv (kD\sin\alpha)^2$$
$$t \equiv 2kD\cos\alpha = 2a_2$$
$$u \equiv h - fD(1-\cos\alpha)$$

Figure 3: Parameters for Vertical Cylindrical Tanks with Conical, Ellipsoidal, or Spherical Bottoms

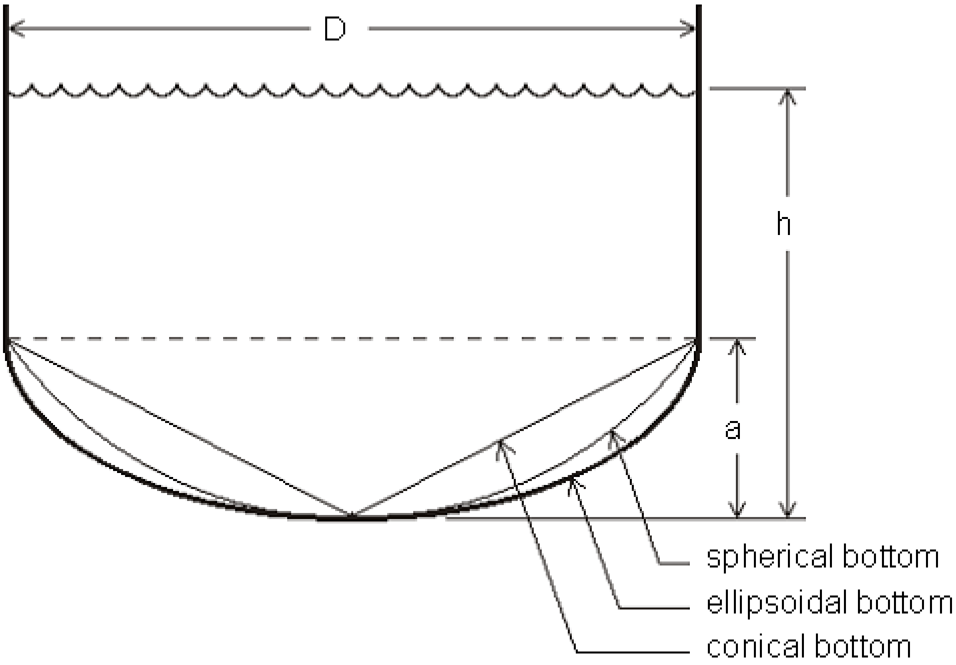
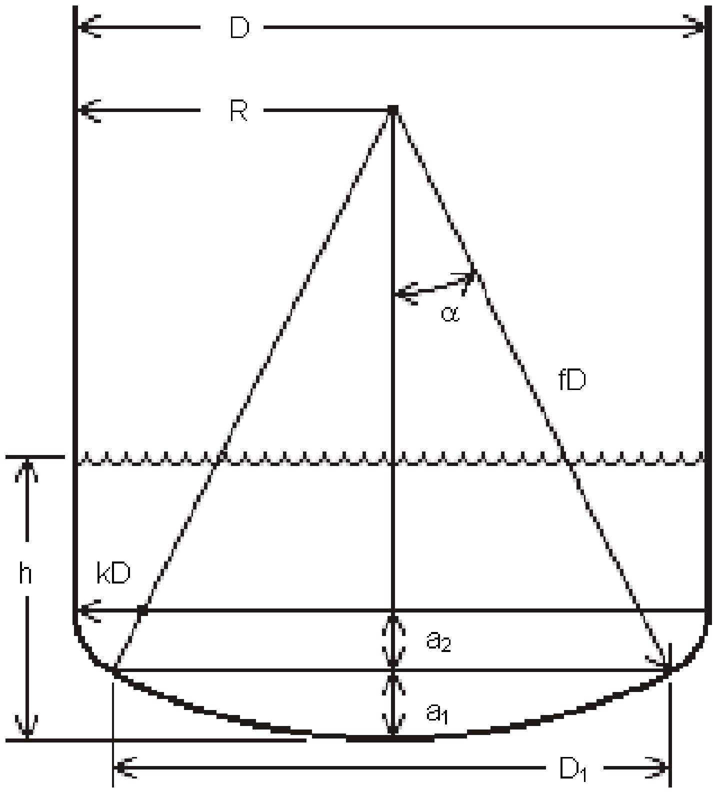


Figure 4: Parameters for Vertical Cylindrical Tanks with Torispherical Bottoms



Nomenclature

a is the distance a horizontal tank's heads extend beyond ($a > 0$) or into ($a < 0$) its cylindrical section or the depth the bottom extends below the cylindrical section of a vertical tank. For a horizontal tank with flat heads or a vertical tank with a flat bottom $a = 0$.

A_f is the cross-sectional area of the fluid in a horizontal tank's cylindrical section.

D is the diameter of the cylindrical section of a horizontal or vertical tank.

D_H, D_W are the height and width, respectively, of the ellipse defining the cross section of the body of a horizontal elliptical tank.

D_A, D_B are the major and minor axes, respectively, of the ellipse defining the cross section of the body of a vertical elliptical tank.

f is the dish-radius parameter for tanks with torispherical heads or bottoms; fD is the dish radius.

h is the height of fluid in a tank measured from the lowest part of the tank to the fluid surface.

k is the knuckle-radius parameter for tanks with torispherical heads or bottoms; kD is the knuckle radius.

L is the length of the cylindrical section of a horizontal tank.

R is the radius of the cylindrical section of a horizontal or vertical tank.

r is the radius of a spherical head for a horizontal tank or a spherical bottom of a vertical tank.

V_f is the fluid volume, of fluid depth h, in a horizontal or vertical cylindrical tank.

Examples – Horizontal Vessels

The following examples were taken from “Computing Fluid Tank Volumes” Chemical Processing, November 17, 2002.

Description:

Find the volumes of fluid, in horizontal cylindrical tanks 108" in diameter with cylinder lengths of 156", with conical, ellipsoidal, guppy, spherical, and “standard” ASME torispherical (f = 1, k = 0.06) heads, each head extending beyond the ends of the cylinder 42" (except torispherical).

D = 108”
L = 156”
a = 42”
k = 0.06
f = 1

Fluid depth = 36”
Fluid depth = 84”

Solution:

D = 2.742 m
L = 3.962 m
a = 1.067 m

Fluid depth, h = 36” = 0.914 m
Fluid depth, h = 84” = 2.134 m

Calculator results:

	Total Liquid Volume (m3)				
Fluid depth (m)	**Conical**	**Elliptical**	**Guppy**	**Spherical**	**Torispherical**
0.914	7.719	9.005	7.305	8.714	7.671
2.134	23.388	26.882	22.533	26.246	22.476

Published results:

	Total Liquid Volume (m3)				
Fluid depth (m)	**Conical**	**Elliptical**	**Guppy**	**Spherical**	**Torispherical**
0.9144	7.727	9.013	7.312	8.721	7.679
2.1336	23.396	26.889	22.539	26.252	22.485

Differences in results are just rounding errors in conversion from imperial to SI units.

Vessel Volume Calculator – Horizontal Vessel Screenshot:

INPUTS

Cylinder length	L	3.962	m
Vessel diameter	D	2.742	m
Vessel head depth	a	1.067	m
Liquid height	h	2.134	m
Vessel orientation		Horizontal	
Vessel head type		Conical	
Knuckle radius parameter	k	0.06	
Dish radius parameter	f	1	

OUTPUT

Total liquid volume in vessel	**23.388**	**m3**

Examples – Vertical Vessels

The following examples were taken from “Computing Fluid Tank Volumes” Chemical Processing, November 17, 2002.

Description:

Find the volumes of fluid, in vertical cylindrical tanks 132" in diameter, 156” in length, with conical, ellipsoidal, spherical, and “standard” ASME torispherical (f = 1, k = 0.06) heads, each head extending beyond the ends of the cylinder 33" (except torispherical).

D = 133”
L = 156”
a = 33”
k = 0.06
f = 1

Fluid depth = 24”
Fluid depth = 60”

Solution:

D = 3.353 m
L = 3.962 m
a = 0.838 m

Fluid depth, h = 24” = 0.610 m
Fluid depth, h = 60” = 1.524 m

Calculator results:

	Total Liquid Volume (m3)			
Fluid depth (m)	**Conical**	**Elliptical**	**Spherical**	**Torispherical**
0.61	0.951	2.969	2.213	3.426
1.524	8.524	10.990	10.065	11.497

Published results:

	Total Liquid Volume (m3)			
Fluid depth (m)	**Conical**	**Elliptical**	**Spherical**	**Torispherical**
0.6096	0.949	2.965	2.217	3.422
1.524	8.522	10.988	10.063	11.495

Differences in results are just rounding errors in conversion from imperial to SI units.

Vessel Volume Calculator – Vertical Vessel Screenshot:

INPUTS

Cylinder length	L	3.962	m
Vessel diameter	D	3.353	m
Vessel head depth	a	0.838	m
Liquid height	h	0.610	m
Vessel orientation		Vertical	
Vessel head type		Conical	
Knuckle radius parameter	k	0.06	
Dish radius parameter	f	1	

OUTPUT

Total liquid volume in vessel	**0.951**	**m3**